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Inflation and Overbanking*

Alexandre Correa Bassoli[†] and Samuel de Abreu Pessoa[‡]

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Abstract

This paper argues that monetary models can and usually present the phenomenon of over-banking; that is, the market solution of the model presents a size of the banking sector which is higher than the social optima. Applying a two sector monetary model of capital accumulation in presence of a banking sector, which supplies liquidity services, it is shown that the rise of a tax that disincentives the acquisition of the banking service presents the following impacts on welfare. If the technology is the same among the sectors, the tax increases welfare; otherwise, steady-state utility increase if the banking sector is labor-intensive compared to the real sector. Additionally, it is proved that the elevation of inflation has the following impact on the economy's equilibrium: the share on the product of the banking sector increases; the product and the stock of capital increases or reduces whether the banking sector is capital-intensive or labor-intensive; and, the steady-state utility reduces. The results were derived under a quite general set up - standard hypothesis regarding concavity of preference, convexity of technology, and normality of goods - were required.

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1 Introduction

The economies which have experienced high level-inflation for a long period of time, also known as chronic inflated economies, have witnessed the increase of the share of the banking sector in the product. As an example, in 1993 Brazil, it skyrocketed to 13% of GDP. Although it is possible that problems exists in Brazilian national accounts, this number is extremely high, especially remembering that for a stabilized economy, this figure is normally around 4%. The comparison gets even worse once it is acknowledged that in a chronic-inflated economy the banking sector does not perform the task of being an intermediate between savers and investors, which is one of its most important duties in a stabilized market economy.

The first objective of this paper is to establish what the real effects of chronicle inflation are. Applying a two-sector version of Sidrauski's (1967) model, which contemplates a banking sector that employing capital and labor offers a money-substitute service, it is shown that a permanent augmentation in the increase rate of the nominal quantity of money has the following impact for the steady-state model's equilibrium: the household utility decreases; the share in the product of the banking sector increases; the money demand decreases, and the banking service demand increases; moreover, the capital stock and the product increase or decrease whether the banking sector is capital-intensive or labor-intensive. These results were derived under a quite general set up - standard hypothesis regarding concavity of preference, convexity of technology, and normality of goods - were required.

The second objective of the paper is to investigate if the market solution for this model presents 'too much banking.' In other words, we are interested to know if the increase of the banking sector in succession of an increase in the inflation rate is a second best policy, **at given inflation rate**. The impact on welfare of a tax levied on the banking-service purchase was studied in order to accomplish this aim. It is shown that the marginal impact of this tax is welfare improving if the technology among the sectors is the same, which means that there is overbanking in this situation.

Aiyagari, Braun, and Eckstein (1998) and English (1999) built a cash-in-advance model in presence of credit goods. In those works is supposed that the acquisition of credit goods is a resource-consumption activity, which is performed by a second sector - the banking sector. In this aspect this paper is complementary to these two works; we develop a Sidrauski-type model and they a cash-in-advance model. The major concern in this paper is generality; in particular, no supposition regarding capital intensity among sector is done. Additionally,

this paper concentrates in the allocation effects of inflation. Even when inflation alters the capital stock, the channel works through allocations impacts under factor prices, and not due to a direct impact of inflation on capital remuneration, as usual in models in which the cash-in-advance restriction applies to capital acquisition.¹ In this aspect this paper puts forth another channel for long-run non-neutrality of money, which is a descendent of Tobin's effect. Another main distinction is that Aiyagari's *et alii* main concern is the impact of inflation on welfare; English's is the impact of inflation on the size of the banking sector; ours is overbanking. In particular, we show that their models present overbanking.

The paper has the following organization. The subsequent Section to this introduction presents the general version of Sidrauski model which contemplates a banking sector. The third Section analyses the short-run equilibria, and the fourth Section derives the long-run comparative-statics results. The ensuing Section investigates the phenomenon of overbanking and the conclusion follows.

2 The General Model

We are interested in investigating the banking sector as a provider of liquidity instruments. The banking institutions are firms that employ capital and labor and supply a service, which are substitutes for cash. Following Sidrauski (1967), we suppose that instantaneous utility depends on consumption and leisure. Leisure is increasing in consumption of liquidity services. We assume that there are two liquidity instruments: in addition to money holding, there is another service, called banking services, which increases household's leisure.

When inflation increases two sort of adjustment are possible. First, the banking sector increases in order to provide liquid services to the public. Second, transaction takes more time; and, this higher time cost could imply a reduction of leisure or of time supply to the labor market. Actually, both margins are present. In this paper we stress the margin between leisure and transaction time; we assume an exogenous supply of labor effort. This supposition localizes all the pecuniary cost of inflation in the extraction of production factor into the banking sector.

In this model the interplay between inflation and factor allocation is as follows: Whenever inflation increases, the private price of money increases, such that the household, in order

¹See Stockman (1981).

to save resources, reduces real money holdings and increases banking services purchases. The increase in banking services demand increases its relative price. In order to equilibrate markets, production factors leave the real sector into the banking sector; consequently, the goods offer reduces and the banking services offer increases. On the other hand, this price movement alters factors prices, producing an impact on capital accumulation.

Households

The choice problem of the household is the following

$$\max \int_0^\infty e^{-\rho t} u(c_{1t}, l_t) dt, \quad (1)$$

where u is the instantaneous utility, which is a function of consumption and leisure. Leisure depends on money holdings, m_t , and consumption of liquidity services, c_{2t} . The household has two units of time; one she inelastically supply to the market, the other she distributes among leisure and transaction, according to

$$l_t \equiv 1 - g(m_t, c_{2t}). \quad (2)$$

Households accumulates assets according to the excess of income, from assets, $r_t a_t$, and wages, w_t ; over purchase of consumption goods, banking services, and cash services. It follows that

$$\dot{a}_t = r_t a_t + w_t + \chi_t - c_{1t} - (1 + \tau)p_t c_{2t} - (\pi_t + r_t)m_t, \quad (3)$$

$$m_t \equiv \frac{M_t}{P_{1t}}, \text{ and } p_t \equiv \frac{P_{2t}}{P_{1t}}, \quad (4)$$

$$a_t \equiv k_t + m_t, \quad (5)$$

where M_t is the nominal *per capita* money stock; P_{1t} is the nominal price of the first good (which could be consumed or accumulated as capital); P_{2t} is the nominal price of the banking service; k_t is the *per capita* capital stock; π_t is the inflation rate; τ is a purchase tax on banking service; and, χ_t is the transfer from the government.

The utility is supposed to be increasing on its arguments, to be strictly concave, and to

satisfy standard Inada conditions. The function g is convex and satisfies

$$g_i < 0, \quad i = 1, 2 \quad (6)$$

$$g_{ii} > 0, \quad i = 1, 2 \text{ and } g_{12} > 0, \quad (7)$$

$$\lim_{m, c_2 \rightarrow \infty} g(m_t, c_{2t}) = 1.$$

The second condition in (7) ensures that money holdings and banking-services consumption are substitutes.²

The first-order condition for the control's variables: consumption, money holding, and banking services are, respectively

$$u_1(c_1, 1 - g(m, c_2)) = \lambda, \quad (8)$$

$$-u_2(c_1, 1 - g(m, c_2))g_1(m, c_2) = \lambda(\pi + r), \quad (9)$$

$$-u_2(c_1, 1 - g(m, c_2))g_2(m, c_2) = \lambda(1 + \tau)p, \quad (10)$$

where λ is the shadow price of income.

The Euler equation for this maximization problem is

$$\frac{\dot{\lambda}}{\lambda} = \rho - r. \quad (11)$$

Firms

Along with the real sector, which produces a good that could be consumed or stocked as capital, there is another sector, called banking sector. This sector produces a service, called banking service, which, as was seen, helps the consumer to cope with inflation. The household supplies inelastically capital and labor force to the market. The final goods and the factors markets are competitive, which make the production side of the model a standard two-sector

²Additionally, we suppose that the functions u and g are of class \mathfrak{C}^2 , such that the solution path of problem (1) are differentiable (Oniki, (1973)).

static model. The following equations summarize the production side of the model³

$$y_1 = l_1 f_1(k_1), \quad (12)$$

$$y_2 = l_2 f_2(k_2), \quad (13)$$

$$l_1 + l_2 = 1, \quad (14)$$

$$k_1 l_1 + k_2 l_2 = k, \quad (15)$$

$$r = f'_1(k_1) \quad (16)$$

$$= p f'_2(k_2), \quad (17)$$

$$w = f_1(k_1) - k_1 f'_1(k_1) \quad (18)$$

$$= p(f_2(k_2) - k_2 f'_2(k_2)), \quad (19)$$

where

$$l_i \equiv \frac{L_i}{L} \text{ and } k_i \equiv \frac{K_i}{L_i},$$

and f_i is per worker product in the i -th sector; L_i is the labor force allocated to the i -th sector; and K_i is the capital stock allocated to the i -th sector.

As usual, standard Inada conditions are assumed:

$$f_i(0) = 0, f'_i > 0, f''_i < 0, \quad (20)$$

$$f'_i(0) = \infty \text{ and } f'_i(\infty) = 0.$$

After solving the eight equations, one is left with the sector's offer function

$$y_1(p, k) = \frac{k_2(p) - k}{k_2(p) - k_1(p)} f_1(k_1(p)), \quad (21)$$

$$y_2(p, k) = \frac{k - k_1(p)}{k_2(p) - k_1(1)} f_2(k_2(p)),$$

³See, for example, Kemp (1969) chapter 1.

in which⁴

$$k'_i \geq 0 \text{ } i = 1, 2 \text{ whether } k_1(p) \geq k_2(p). \quad (22)$$

The comparative static for these functions are the following:

$$y_{11} \equiv \frac{\partial y_1}{\partial p} \Big|_k < 0, \text{ and } y_{12} \equiv \frac{\partial y_2}{\partial p} \Big|_k > 0, \quad (23)$$

$$y_{12} \equiv \frac{\partial y_1}{\partial k} \Big|_p \geq 0 \text{ and } y_{22} \equiv \frac{\partial y_2}{\partial k} \Big|_p \leq 0 \text{ whether } k_1 \geq k_2. \quad (24)$$

Additionally, it is known from the inclination of the production possibility curve that

$$\frac{\partial y_1}{\partial p} \Big|_k + p \frac{\partial y_2}{\partial p} \Big|_k = 0, \quad (25)$$

and from the marginal impact of capital that

$$\frac{\partial y_1}{\partial k} \Big|_p + p \frac{\partial y_2}{\partial k} \Big|_p = \frac{\partial}{\partial k} (y_1 + p y_2) \Big|_p = r. \quad (26)$$

3 Short-Run Equilibrium

The banking service is a non-storable good, and, consequently, its market clears continuously. The banking-services relative price adjusts in order to accomplish the equilibria. Formally,

$$c_{2t} = y_2(p_t, k_t). \quad (27)$$

Substituting the market-clear condition for the banking sector into the first-order condition, one is left with the equation system which describes the short-run, or momentary,

⁴This is a basic result in two-sector models. In words: if the banking sector is labor-intensive, an increase of the banking-service's relative price provokes an inflow of production factors towards it, which will cause an increase in the relative remuneration of labor, as comparing to the remuneration of capital, in order to equilibrate the factor market. This increase in the labor cost will engender an increase in the capital intensity by the firms. An opposite argument applies for the situation in which the banking sector is capital-intensive.

equilibrium of this economy. They are

$$u_1(c_1, 1 - g(m, y_2(p, k))) = \lambda, \quad (28)$$

$$-u_2(c_1, 1 - g(m, y_2(p, k)))g_2(m, y_2(p, k)) = (1 + \tau)\lambda p, \quad (29)$$

and

$$\frac{g_1(m, y_2(p, k))}{g_2(m, y_2(p, k))} = \frac{\pi + f'_1(k_1(p))}{(1 + \tau)p}. \quad (30)$$

In the last equation (16) was substituted. The system is separable: (28) and (29) solves for c_1 and p as function of the state variable k , the costate-variable λ , the costate-like variable m , and the exogenous variable τ . Subsequently, (30) solves for π . Table 1 displays the static-comparative results.⁵

		Table 1		k	m	λ	τ
		$k_1 > k_2$	$k_1 < k_2$				
c_1	$u_{12} > 0$	—	+	+	—	—	
	$u_{12} < 0$	+	—	—	—	+	
p		+	—	—	—	—	

The intuition for these movements are straightforward. For example, an increase in the capital stock will increase the supply, and, consequently, the consumption of the second good, if the second sector is capital-intensive, resulting in a decrease of the marginal utility of the banking-service. To restore the equilibrium the price of the banking-service will decrease, reducing its production; and the consumption of the first good will increase or decrease, whether it is a complement or a substitute for leisure. This explains the fourth column of the table. Analogous histories could be told respecting the other columns. The comparative-static for the inflation rate is indeterminate when the banking sector is capital-intensive because, under that hypothesis, the relative price and the real interest rate change in the same direction, making impossible, at this generality level, to signal it.⁶ However, as it

⁵Details in the appendix 2.

⁶Follows directly from (30).

will be clear in the next section, this indeterminacy will not cause any harm for the long-run comparative-static analysis. The details of the calculations are presented in the appendix 2.

The government's budget constraint is

$$\chi = \sigma m + \tau p c_2, \quad (31)$$

in which $\sigma \equiv \frac{\dot{M}}{M}$. The money market equilibrium equation is

$$\dot{m} = \sigma m - \pi m. \quad (32)$$

After substituting (31) and (32) into (3) and recalling (5), it follows that

$$\dot{k} = w + rk - c_1 - p c_2. \quad (33)$$

For this closed economy, *per capita* income is equal to *per capita* product, which means that⁷

$$w + rk = y_1 + p y_2;$$

which, together with the second-sector market equilibrium equation, implies that

$$\dot{k} = y_1 - c_1. \quad (34)$$

The dynamic system formed by (11), (32), and (34) describes the evolution of λ , m , and k .

⁷It follows from

$$\begin{aligned} w + rk &= (f_1 - k_1 f'_1) + f'_1 k \\ &= (f_1 - k_1 f'_1)(l_1 + l_2) + f'_1(l_1 k_1 + l_2 k_2) \\ &= l_1 f_1 + p(f_2 - k_2 f'_2)l_2 + p f'_2 l_2 k_2 \\ &= l_1 f_1 + p l_2 f_2 \\ &= y_1(p, k) + p y_2(p, k). \end{aligned}$$

In which the equations (12)-(19) were employed.

4 Long-Run Analysis

We are interested in evaluating the long-run impact of a changing in the increase rate of the nominal quantity of money on: capital, money holdings, consumption, banking-sector output, banking-sector share in output, *per capita* income, and welfare. For the steady-state, it follows that

$$y_1(p(k, \lambda, m, \tau), k) - c_1(k, \lambda, m, \tau) = 0, \quad (35)$$

$$\rho - f'_1(k_1(p(k, \lambda, m, \tau))) = 0, \quad (36)$$

$$\sigma - \pi(k, \lambda, m, \tau) = 0. \quad (37)$$

Although, as will be seen, the capital stock is not invariant with respect to inflation rate, this economy presents a weak neutrality - the long-run interest rate is not dependent of inflation. This property distinguishes this model from cash-in-advance models in the presence of credit goods, when the cash-in-advance restriction applies to capital good. In that class of models the inflation acts like a distortion on capital income,⁸ hindering its accumulation. Here the long-run remuneration of capital, and, consequently, the relative price of banking services, is not dependent on inflation. From this point of view, there is not a direct impact of inflation on capital's remuneration. What occurs is that the inflation rate changes the long-run banking services output. When inflation increases, the household substitutes banking-service for money. Because in the long-run the relative price does not change,⁹ the unique way to increase the banking-sector output is to change the factor endowment of this economy,¹⁰ which means capital accumulation if the banking sector is capital-intensive, or capital reduction if the opposite occurs.

⁸This observation is subjected to qualifications. See Pessoa (2000).

⁹This result follows from (36).

¹⁰In the International Trade literature this result is known as Rybczynski-Samuelson theorem.

Differentiating totally the equation system, it follows that

$$\begin{aligned} & \begin{bmatrix} y_{11}p_1 + y_{12} - c_{11} & y_{11}p_2 - c_{12} & y_{11}p_3 - c_{13} \\ -f_1''k_1'p_1 & -f_1''k_1'p_2 & -f_1''k_1'p_3 \\ -\pi_1 & -\pi_2 & -\pi_3 \end{bmatrix} \begin{bmatrix} dk \\ d\lambda \\ dm \end{bmatrix} \\ &= - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} d\sigma + \begin{bmatrix} c_{14} - y_{11}p_4 \\ f_1''k_1'p_4 \\ \pi_4 \end{bmatrix} d\tau. \end{aligned} \quad (38)$$

Recalling that the functions describing p , c_1 , and π came from a short-run static-comparative, and substituting into (38) its derivatives, which were calculated in the second appendix, the Jacobian of the long-run equilibrium follows¹¹

$$\begin{aligned} \Lambda &= -\frac{f_1''k_1'}{\Delta^3} \frac{1}{g_2^7 p^2} (-y_{21}g_2^3 |u_{ij}| + u_{11}\lambda(g_2 - g_{22}py_{21}))^2 \\ &\quad \left\{ y_{22} \left(p\lambda |g_{ij}| - (pu_{12} + g_2 u_{22}) \begin{vmatrix} g_{ij} & g_i \\ g_j & 0 \end{vmatrix} \right) \right. \\ &\quad \left. + y_{12}(pu_{11} + g_2 u_{21})(g_1 g_{21} - g_2 g_{11}) \right\}, \end{aligned} \quad (39)$$

in which

$$|u_{ij}| \equiv u_{11}u_{22} - u_{12}u_{21} > 0, \quad (40)$$

$$|g_{ij}| \equiv g_{11}g_{22} - g_{12}g_{21} > 0, \quad (41)$$

$$\begin{vmatrix} g_{ij} & g_i \\ g_j & 0 \end{vmatrix} \equiv 2g_1g_2g_{12} - g_1^2g_{22} - g_2^2g_{11} < 0,$$

and

$$\Delta \equiv -g_2y_{21}|u_{ij}| + \frac{u_{11}\lambda}{g_2} \left(1 - \frac{pg_{22}y_{21}}{g_2}\right) > 0 \quad (42)$$

¹¹The reader might be wondering how we perform this calculation. In fact, each entry of the Jacobian came from a short-run comparative statics, which involves three equations. Which means that this determinant is 9×9 . To calculate the Jacobian and the others determinants we first simplified the expression with the help of the mathematica software.

is the Jacobian of the short-run comparative-statics.¹²

Proposition 1 *Under the following hypothesis the Jacobian evaluated at the steady-state is **negative**: utility and leisure are strictly concave function of theirs arguments; the consumption good and leisure are normal goods; money holds and banking-service consumption are normal goods; and the offer-functions are represented by (21).*

Proof. It follows directly from (20), (22), (23)-(24), (40)-(42), (51)-(54). ■

Once the Jacobian is signalized we can employ Cramer's rule to evaluate the long-run impact of a change in σ on k , λ , m . Let

$$\begin{aligned}\Lambda_{\sigma}^k &\equiv \begin{vmatrix} 0 & y_{11}p_2 - c_{12} & y_{13}p_3 - c_{13} \\ 0 & -f_1''k_1'p_2 & -f_1''k_1'p_3 \\ -1 & -\pi_2 & -\pi_3 \end{vmatrix} \\ &= -\frac{f_1''k_1'}{\Delta^2} \frac{1}{g_2^4p^2} [g_1g_2(pu_{12} + g_2u_{22}) + g_{21}p\lambda] \\ &\quad (-g_2^3y_{21}|u_{ij}| + g_2u_{11}\lambda - pg_{22}u_{11}y_{21}\lambda) \\ &\geq 0 \text{ whether } k_1 \geq k_2.\end{aligned}\tag{43}$$

In the same way

$$\begin{aligned}\Lambda_{\sigma}^m &\equiv \begin{vmatrix} y_{11}p_1 + y_{12} - c_{11} & y_{11}p_2 - c_{12} & 0 \\ -f_1''k_1'p_1 & -f_1''k_1'p_2 & 0 \\ -\pi_1 & -\pi_2 & -1 \end{vmatrix} \\ &= \frac{f_1''k_1'}{\Delta^2} \frac{1}{g_2^4p^2} [-g_2^3y_{21}|u_{ij}| + u_{11}\lambda(g_2 - g_{22}py_{21})] \\ &\quad [y_{22}(g_2^2(pu_{12} + g_2u_{22}) + g_{22}p\lambda) - y_{12}g_2(pu_{11} + g_2u_{12})] \\ &> 0.\end{aligned}\tag{44}$$

Proposition 2 *Under the hypothesis of proposition 1, after a permanent increase in the growth rate of the nominal quantity of money, the long-run capital stock decreases or increases whether the real sector is capital-intensive or labor-intensive, and the long-run real quantity of money decreases.*

¹²See the second appendix.

Proof. The result follows directly from proposition 1, (43), and (44). ■

Proposition 3 *Under the hypothesis of proposition 1, after a permanent increase in the growth rate of the nominal quantity of money, the consumption of the banking-service increases.*

Proof. From the market equilibrium equation, we know that

$$c_2 = y_2(p, k),$$

which means that

$$\frac{dc_2}{d\sigma} = y_{22} \frac{dk}{d\sigma}.$$

The result follows from proposition 2 and (24). ■

Proposition 4 *Under the hypothesis of proposition 1, after a permanent increase in the growth rate of the nominal quantity of money, the share of the banking-sector in the product increases.*

Proof. The share of the banking-sector in the product is

$$\alpha_2 \equiv \frac{py_2}{y_1 + py_2}.$$

Consequently,

$$\frac{d\alpha_2}{d\sigma} = p \frac{y_1 y_{22} - y_2 y_{12}}{(y_1 + py_2)^2} \frac{dk}{d\sigma}.$$

The result follows from proposition 2 and (24). ■

The impact of σ on the shadow price of consumption, λ , is indeterminate. The reason is that, as will be seen, consumption and leisure always decrease after an increase in σ . If consumption and leisure are substitutes, these movements act in the direction of increasing the marginal utility of consumption; on the other hand, if they are complements, the impact

on the marginal utility of consumption of an increase of σ is indeterminate. Formally,

$$\begin{aligned}
\Lambda_{\sigma}^{\lambda} &= \begin{vmatrix} y_{11}p_1 + y_{12} - c_{11} & 0 & y_{13}p_3 - c_{13} \\ -f_1''k_1'p_1 & 0 & -f_1''k_1'p_3 \\ -\pi_1 & -1 & -\pi_3 \end{vmatrix} \\
&= \frac{f_1''k_1'}{\Delta^2} \frac{1}{g_2^4 p^2} [-g_2^3 y_{21} |u_{ij}| + u_{11} \lambda (g_2 - g_{22} p y_{21})] \\
&\quad [y_{12} (-u_{11} g_{21} p \lambda - g_1 g_2^2 |u_{ij}|) + y_{22} u_{12} p \lambda (g_2 g_{21} - g_1 g_{22})] \\
&< 0 \text{ if } u_{12} \leq 0 \text{ and undetermined if } u_{12} > 0.
\end{aligned} \tag{45}$$

The next proposition investigates the impact on consumption, leisure, and utility.

Proposition 5 *Under the hypothesis of proposition 1, after a permanent increase in the growth rate of the nominal quantity of money, the following adjustment take place: consumption, leisure, and utility decrease.*

Proof.

1. Consumption. The total impact of an alteration of σ on consumption is given by

$$\frac{dc_1}{d\sigma} = \frac{\partial c_1}{\partial k} \frac{dk}{d\sigma} + \frac{\partial c_1}{\partial \lambda} \frac{d\lambda}{d\sigma} + \frac{\partial c_1}{\partial m} \frac{dm}{d\sigma}.$$

Substituting (56)-(58), (43)-(45), it follows that

$$\begin{aligned}
\frac{dc_1}{d\sigma} &= -\frac{f_1''k_1'}{\Delta^3 \Lambda} \frac{1}{g_2^6 p^3} [-g_2^3 y_{21} |u_{ij}| + u_{11} \lambda (g_2 - g_{22} p y_{21})]^2 \\
&\quad y_{12} [g_1 g_2 (p u_{12} + g_2 u_{22}) + g_2 u_{22}].
\end{aligned}$$

The result follows directly from the hypothesis.

2. Leisure. Following the same steps, one gets

$$\begin{aligned}
\frac{dl}{d\sigma} &= -\frac{f_1''k_1'}{\Delta^3 \Lambda} \frac{1}{g_2^6 p^3} [-g_2^3 y_{21} |u_{ij}| + u_{11} \lambda (g_2 - g_{22} p y_{21})]^2 \\
&\quad [y_{22} p \lambda (g_2 g_{21} - g_1 g_{22}) + y_{12} g_1 g_2 (p u_{11} + g_2 u_{21})].
\end{aligned}$$

The result follows directly from the hypothesis.

3. Utility. Following the same steps, one gets

$$\begin{aligned} \frac{du}{d\sigma} &= \frac{f_1'' k_1'}{\Delta^3 \Lambda g_2^7 p^3} \left[-g_2^3 y_{21} |u_{ij}| + u_{11} \lambda (g_2 - g_{22} p y_{21}) \right]^2 \\ &\quad [y_{22} p^2 \lambda (g_2 g_{21} - g_1 g_{22}) \\ &\quad + y_{12} (g_2 g_{21} p \lambda + p g_1 g_2 (p u_{11} + g_2 u_{21}) + g_1 g_2^2 (p u_{21} + g_2 u_{22}))]. \end{aligned}$$

The result follows directly from the hypothesis.

■

Having established the main long-run results regarding the interaction between money demand, banking sector, and inflation, let's turn to the second objective of the paper, which is to study the optimum size of the banking sector.

5 Overbanking

As was seen in the last section, the increase of the inflation rate fosters the increase of the banking sector; its share in the product increases. Our concern here is to evaluate if this phenomenon is optimum from the social point of view. On one hand, the attraction of production factors from the real sector eases the provision of money-substitutes liquidity-services, and, consequently, helps the public to cope with inflation. On the other hand, these production factors have a social opportunity cost given by its shadow price in the real sector. It is not clear how those factors trade-off. To address this issue, we evaluate the impact on welfare and long-run utility of a purchase tax levied on the banking service.

From (38) it is possible to evaluate the long-run impact of the purchase tax on money holdings and capital stock. It follows that,

$$\begin{aligned} \Lambda_\tau^k &= \begin{vmatrix} c_{14} - y_{11} p_4 & y_{11} p_2 - c_{12} & y_{13} p_3 - c_{13} \\ f_1'' k_1' p_4 & -f_1'' k_1' p_2 & -f_1'' k_1' p_3 \\ \pi_4 & -\pi_2 & -\pi_3 \end{vmatrix} \\ &= -\frac{f_1'' k_1'}{\Delta^3 g_2^7 p^2} (-y_{21} g_2^3 |u_{ij}| + u_{11} \lambda (g_2 - g_{22} p y_{21}))^2 \\ &\quad [p^2 \lambda g_2 g_{11} + g_1^2 g_2 p (p u_{12} + g_2 u_{22})] \\ &\geq 0 \text{ whether } k_1 \geq k_2, \end{aligned} \tag{46}$$

and

$$\begin{aligned}
\Lambda_{\tau}^m &= \begin{vmatrix} y_{11}p_2 - c_{12} & c_{14} - y_{11}p_4 & y_{13}p_3 - c_{13} \\ -f_1''k_1'p_2 & f_1''k_1'p_4 & -f_1''k_1'p_3 \\ -\pi_2 & \pi_4 & -\pi_3 \end{vmatrix} \\
&= -\frac{f_1''k_1'}{\Delta^3} \frac{1}{g_2^7 p^2} (-y_{21}g_2^3 |u_{ij}| + u_{11}\lambda(g_2 - g_{22}py_{21}))^2 \\
&\quad [y_{22}(g_2^2 pg_1(pu_{12} + g_2u_{22}) + p^2\lambda g_2g_{12}) \\
&\quad - y_{12}pg_1g_2(pu_{11} + g_2u_{21})] \\
&< 0.
\end{aligned} \tag{47}$$

Then, we establish:

Proposition 6 *Under the hypothesis of proposition 1, after a permanent increase in the purchase tax, the long-run capital stock increases or decreases whether the banking sector is labor-intensive or capital-intensive, and the real quantity of money increases.*

Proof. It follows from proposition 1, (46), and (47). ■

A straightforward corollary of this last proposition is that the banking-sector output and its share in the product decreases after a marginal increase in the purchase tax.

The next step is to evaluate the impact of the purchase tax on long-run utility. After recalling the long-run goods equilibrium equation, the first-order conditions, and (26), it follows that

$$\left. \frac{du}{d\tau} \right|_{\tau=0} = u_1 \left(r \left. \frac{dk}{d\tau} \right|_{\tau=0} + (\pi + r) \left. \frac{dm}{d\tau} \right|_{\tau=0} \right) > 0 \text{ if } k_1 \geq k_2.$$

There are two effects. Firstly, the elevation in long-run money holdings, which takes place after a marginal increase in tax, increases long-run utility. Secondly, the impact on long-run utility of the adjustment of capital. If the first sector is capital-intensive, or, if both share the same technology, those two effects acts in the same direction; otherwise, is not possible to signal the impact on long-run utility of the marginal increase on the purchase tax. The reason is that the reduction of capital stock brought about by the decrease of the banking sector's output reduces long-run utility. Notwithstanding, this reduction is compensated by

an increase in welfare along the transitory dynamic. Let's evaluate the impact on welfare. In appendix 1.3 it is shown that

$$\frac{dW}{d\tau}\Big|_{\tau=0} = \int_0^\infty e^{-\rho t} \lambda(\pi + r) \frac{dm}{d\tau}\Big|_{\tau=0} dt. \quad (48)$$

For the entire dynamic, the impact of a marginal increase in the purchase tax on welfare, for an initial situation in which the tax is absent, depends only on its impact on the money demand. In particular, the effect on welfare of capital accumulation is zero, as one would expect.¹³ The monetary models are not optimum in Pareto's sense. The result (48) establishes that this model generates trajectories which are Pareto-Optimum restricted¹⁴; that is, given that this economy is using less money services than the social optimum, the other variables, including capital, are at an extremum of the welfare function. From (48) it follows that a sufficient condition for the increase in welfare after an increase in the tax is that the money demand increases along the dynamic path of the economy. Given that the long-run money demand increases, if the first period real quantity of money increases, we know from Bernheim (1981) that the entire path will increase. Although, at this level of generality, it is not possible to know if that is the case. If the technology is the same among sectors, capital accumulation does not take place after the increase in the tax. Under this hypothesis, it follows from (48) that

$$\rho \frac{dW}{d\tau}\Big|_{\tau=0} = \lambda(\pi + r) \frac{dm}{d\tau}\Big|_{\tau=0} > 0. \quad (49)$$

The phenomenon of overbanking occurs. This last expression gives a formal content to Johnson's (1968) statement:

“The substantive point is that, because the private cost of holding currency (the interest forgone) substantially exceeds the social cost (raw material, value added, and policing), free competition in banking, by making the private and social cost of deposit holding coincide, would tend to produce a social non-optimum overallocation of resources to the provision of deposit money and underallocation of resources to the provision of currency holding. (...) On the assumption that

¹³We thank Marcos de Barros Lisboa for this observation.

¹⁴In fact, this is a very general property of monetary models. See Pessoa (2000).

currency cannot be issued other than as a non-interest-bearing asset, achievement of the “second best” welfare optimum would require a tax on the holding of deposit money at a rate somewhere between zero and the competitive interest rate in deposits (...)” (p. 974.)

Is this result an artifact of the Sidrauski version employed here? To answer this question let’s turn to Aiyagari’s *et alii* (1998) and English’s (1999) models. These works examine a cash-in-advance economy in presence of credit goods. There is a continuum of goods, which could be acquired in the market, in exchange for money or a credit service. Under this second possibility, the price of a good is the money price plus a cost which varies, depending on the good.¹⁵ In this class of cash-in-advance models, there is a cut-off index which splits the continuum of goods in those which are acquired by cash and those by credit. The higher the inflation rate, the larger the range of goods acquired by credit is and, consequently, the higher the money velocity is.¹⁶ Similar to the present work, these models take into account that the provision of this money-substitutes services by the banking sector requires the employment of production factors, which have been diverted from the real sector.¹⁷ Let’s first consider the version in which the long-run capital-stock is not sensitive to the inflation rate - that is, in which labor supply is inelastic, and capital is a cash good whose purchase does not present transaction cost.¹⁸

It is straightforward to show that a sufficient high tax eliminates totally the misallocation effect of inflation on welfare. For the Aiyagari *et alii* formulation, the argument goes as follows. There is a continuum of goods. If the household acquires a good in the market as cash, the cost, in units of income, will be $1 + R_t$, in which R stands for the nominal interest rate; if it acquires it as credit good, the cost will be $(1 + \tau)(1 + \mathfrak{R}(z))$, in which: τ stands by the purchase tax on the banking service; \mathfrak{R} stands by the transaction cost function; and

¹⁵Differently, English supposes that there is a weighing function in the preferences which varies across type of goods.

¹⁶This manner of producing a variable money velocity in cash-in-advance models was introduced by Gillman (1993).

¹⁷Aiyagari’s *et alii* (1998) and English’s (1999) models are very similar. The main difference is the modeling of the transaction technology. Aiyagari *et alii* suppose a constant marginal cost in acquiring a good as credit good while English models it as a fixed cost.

¹⁸If capital is fixed, it is possible to allow an endogenous determination of the labor supply.

z by the good's index. The cut-off good is the one which solves

$$1 + R_t = (1 + \tau)(1 + \Re(z_t^*)).$$

As Aiyagari *et alii* stressed, this is a “Baumol-type condition that sets the opportunity cost of cash equal to the cost of credit services for the marginal good.” (p. 1286) If the tax is set equal to R_t , we get that

$$\Re(z_t^*) = 0 \Leftrightarrow z_t^* = 0.$$

Every good is purchased as cash good, and in this economy inflation is harmless! The tax is on the upper end of Johnson's range and ‘fixes’ perfectly the economy; the welfare is the same as under Friedman's rule, although inflation is positive. The second-best solution provides the same welfare as the first-best. A similar argument, with adaptations, applies to English's model. Consequently, this class of models presents a very strong version of overbanking. This strong result we do not get in the Sidrauski formulation. Due to the nature of the $g(m, c_2)$ function, at any inflation rate the banking sector can do something for which cash is not a perfect substitute. Saying differently, in the Sidrauski model at any inflation level there is an optimum size of the banking sector, which is higher than zero; the point here is that the market solution usually induces an extent larger than this optimum size.

This extreme form of overbanking for this class of cash-in-advance models takes place because, from the social point of view, the banking institutions are useless. One way of giving a social task to banking in this environment is to suppose that the cash-in-advance restriction applies to capital purchase. In a short and influential paper, Stockman (1981) showed that capital accumulation is harmed in a cash-in-advance economy, if capital's purchase is subjected to the cash-in-advance restriction. The shadow price of the cash-in-advance restriction acts like a distorted tax on investment, reducing long-run capital stock. Formally, the shadow price of capital is

$$\lambda + \mu = \lambda(1 + R),$$

in which μ stands for the shadow price of cash. Therefore, after an increase on inflation, and consequently of the nominal interest rate, the economy de-accumulates capital. When there

is a continuum of goods, as we saw, every good whose transaction cost in the credit sector is lower than R is acquired as credit good, paying, obviously, the cost of the banking service. Hence, the transaction cost is a ceiling to the distortion effect of inflation. From this point of view the increase in the banking sector is a good.

Now, there are two impacts of inflation on the equilibrium of the economy. First, the diversion of production factors from the real sector towards the banking sector - the misallocation effect. Second, the reduction in the capital stock which takes place after an increase in the inflation rate - the distortion effect. Elsewhere, Pessoa (2000) proved that for this class of models it is true that

$$\frac{dW}{d\sigma} = \sum_{t=0}^{\infty} \beta^t \lambda_t R_t \frac{dm_t}{d\sigma},$$

which means that capital is optimal-chosen from the social point of view. In other words, variations in the capital-stock path produce second-order effects on the welfare function, and, consequently, the distortion effect is second-order small. Because the misallocation effect is first-order, this economy presents the overbanking phenomenon. For that reason, it is optimum to impose a tax on banking service acquisition, but, probably is not optimum to drive off the banking sector. This is exactly the sort of overbanking that was obtained with the Sidrauski formulation.

6 Conclusion

The main conclusion is that the existence of banking reinforces Friedman's (1969) result. Inflation harm the economy because household carries too small money stock, and, consume too much banking services. Due to the second best nature of the monetary economy studied in this paper, an additional distortion increases welfare. However, our approach in this paper was macroeconomic; banking is a firm that employs capital and labor to produce a liquid service, which substitutes cash. There was no room here to some institutional peculiarity of banking, like reserves requirement ratio and restriction to below market remuneration for some assets.

Another point is the robustness of our result to other formulations of preferences or transaction cost functions. For each particular formulation of preference or transaction cost

function it is possible to derive an expression equivalent to (48). Saying differently, the result that monetary models imply trajectories which are Pareto Optimum restricted to a given path of cash out of the banking sector, is a robust result for a competitive banking sector without reserve requirements and other regulations. Consequently, for this class of models, if they additionally have the propriety of long-run capital neutrality, there is overbanking. If the model does not present long-run capital's neutrality, probably, due to the second order nature of the distortion effect of inflation on welfare, it still will show up overbanking.

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1 Appendix

1.1 Normality Conditions

Throughout the paper it was made the hypothesis that the three goods - the good, money, and banking services - were normal goods. This appendix derives the conditions that ensures that this normality conditions are validly.

The first set of normality conditions came form the following static problem:

$$\begin{aligned} & \min g(m, c_2) \\ \text{subject to} \quad & : \quad (\pi + r)m + pc_2 = y. \end{aligned} \tag{50}$$

The second-order conditions for this maximization problem is automatically attained by the convexity of g . Solving the comparative static for the first-order condition, it follows that the money and banking services' consumption increase after an increase in income if and only if

$$g_1g_{22} - g_2g_{12} < 0, \tag{51}$$

$$g_2g_{11} - g_1g_{21} < 0. \tag{52}$$

The second set of normality conditions came from the following maximization problem:

$$\begin{aligned} & \max u(c_1, l) \\ \text{subject to} \quad & : \quad c_1 + ql = y, \end{aligned}$$

in which q is the price in unit of goods of leisure. Analogously, redoing the steps which lead

to (51) and (52) it follow that

$$u_1 u_{22} - u_2 u_{12} < 0,$$

$$u_2 u_{11} - u_1 u_{21} < 0.$$

Recalling (8) and (10) these two conditions could be written as

$$p u_{11} + g_2 u_{12} < 0, \tag{53}$$

$$p u_{21} + g_2 u_{22} > 0. \tag{54}$$

1.2 Short-Run Comparative-Static

In this appendix the short-run comparative-statics is solved. The equation which describes the short-run equilibrium are the first order conditions (8)-(10), and the banking-service's market-equilibrium equation (27). This system of three equations solves for c_1 , p , and π as a function of the state variable (k), the costate variable (λ), the costate-like variable (m), and the exogenous variable (τ). The equations

$$\begin{aligned} u_1(c_1, 1 - g(m, y_2(p, k))) &= \lambda, \\ -u_2(c_1, 1 - g(m, y_2(p, k)))g_2(m, y_2(p, k)) &= \lambda(1 + \tau)p \end{aligned}$$

solves for c_1 and p . Differentiating the system, it follows that

$$\begin{aligned} & \begin{bmatrix} u_{11} & -g_2 u_{12} y_{21} \\ u_{21} & -g_2 u_{22} y_{21} + \frac{\lambda}{g_2^2} (g_2 - g_{22} p y_{21}) \end{bmatrix} \begin{bmatrix} dc_1 \\ dp \end{bmatrix} \\ &= \begin{bmatrix} u_{12} g_1 \\ u_{22} g_1 + \frac{\lambda}{g_2^2} p g_{21} \end{bmatrix} dm + \begin{bmatrix} u_{12} g_2 y_{22} \\ g_2 u_{22} y_{22} + \frac{\lambda}{g_2^2} g_{22} p y_{22} \end{bmatrix} dk \\ &+ \begin{bmatrix} 1 \\ -\frac{p}{g_2} \end{bmatrix} d\lambda - \begin{bmatrix} 0 \\ \frac{\lambda p}{g_2} \end{bmatrix} d\tau. \end{aligned}$$

Applying Cramer's rule, it follows that

$$\Delta = -g_2 y_{21} |u_{ij}| + \frac{\lambda u_{11}}{g_2} (1 - p y_{21} \frac{g_{22}}{g_2}) > 0, \quad (55)$$

$$\Delta_m^{c_1} = u_{12} \frac{\lambda}{g_2^2} [p y_{21} (g_2 g_{21} - g_1 g_{22}) + g_1 g_2] \geq 0 \text{ whether } u_{12} \geq 0, \quad (56)$$

$$\Delta_k^{c_1} = u_{12} y_{22} \lambda, \quad (57)$$

$$\Delta_\lambda^{c_1} = -y_{21} (p u_{12} + g_2 u_{22}) + \frac{\lambda}{g_2} (1 - p y_{21} \frac{p}{g_2}) < 0, \quad (58)$$

$$\Delta_\tau^{c_1} = \lambda p u_{12} y_{21} \geq 0 \text{ whether } u_{12} \geq 0, \quad (59)$$

$$\Delta_m^p = g_1 |u_{ij}| + \frac{\lambda}{g_2^2} p u_{11} g_{21} < 0, \quad (60)$$

$$\Delta_k^p = y_{22} (g_2 |u_{ij}| + \frac{\lambda}{g_2^2} p u_{11} g_{22}) \geq 0 \text{ whether } k_1 \geq k_2, \quad (61)$$

$$\Delta_\lambda^p = -\frac{1}{g_2} (p u_{11} + g_2 u_{21}) < 0, \quad (62)$$

$$\Delta_\tau^p = -\frac{\lambda}{g_2} p u_{11} < 0. \quad (63)$$

From (30) it is possible to calculate the comparative static for the inflation rate.

1.3 Derivation of the Expression (48)

From (1), it follows that

$$\begin{aligned} \frac{dW}{d\tau} &= \int_0^\infty e^{-\rho t} \frac{d}{d\tau} u(c_1, l(m, c_2)) dt \\ &= \int_0^\infty e^{-\rho t} \lambda \left[\frac{dc_1}{d\tau} - (\pi + r) \frac{dm}{d\tau} - p(1 + \tau) \frac{dc_2}{d\tau} \right] dt, \end{aligned} \quad (64)$$

in which the last equality follows after substituting the first-order conditions (8)-(10).

From the equilibrium in the market for goods it is known that

$$\int_0^\infty e^{-\rho t} \lambda \frac{d}{d\tau} \left(y_1(p, k) - c_1 - \dot{k} \right) dt = 0,$$

and from the banking services market

$$\int_0^\infty e^{-\rho t} \lambda p \frac{d}{d\tau} (y_2(p, k) - c_2) dt = 0;$$

which could respectively be written as

$$\int_0^\infty e^{-\rho t} \lambda \left(y_{11} \frac{dp}{d\tau} + y_{12} \frac{dk}{d\tau} - \frac{dc_1}{d\tau} - \frac{\dot{k}}{d\tau} \right) dt = 0, \quad (65)$$

and

$$\int_0^\infty e^{-\rho t} \lambda p \left(y_{21} \frac{dp}{d\tau} + y_{22} \frac{dk}{d\tau} - \frac{dc_2}{d\tau} \right) dt = 0. \quad (66)$$

Integrating by parts the last term in (65) and recalling that capital does not jump and it is bounded, it follows that

$$\int_0^\infty e^{-\rho t} \lambda \frac{d}{dt} \frac{dk}{d\tau} dt = - \int_0^\infty e^{-\rho t} \lambda \left(-\rho + \frac{\dot{\lambda}}{\lambda} \right) \frac{dk}{d\tau} dt. \quad (67)$$

Substituting (67) into (65), adding the result and (66) to (64), it follows that

$$\begin{aligned} \frac{dW}{d\tau} = & \int_0^\infty e^{-\rho t} \lambda \left\{ \left[\frac{dc_1}{d\tau} + (\pi + r) \frac{dm}{d\tau} + p(1 + \tau) \frac{dc_2}{d\tau} \right] \right. \\ & + \left(y_{11} \frac{dp}{d\tau} + y_{12} \frac{dk}{d\tau} - \frac{dc_1}{d\tau} + \left(-\rho + \frac{\dot{\lambda}}{\lambda} \right) \frac{dk}{d\tau} \right) \\ & \left. + p \left(y_{21} \frac{dp}{d\tau} + y_{22} \frac{dk}{d\tau} - \frac{dc_2}{d\tau} \right) \right\} dt. \end{aligned}$$

Recalling (11), (25), and (26), (48) follows.